Title: Effects of a Research-Based Intervention to Improve Seventh-Grade Students' Proportional Problem Solving: A Cluster Randomized Trial

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Abstract

This experimental study evaluated the effectiveness of a research-based intervention, schema-based instruction (SBI), on students' proportional problem solving. SBI emphasizes the underlying mathematical structure of problems, uses schematic diagrams to represent information in the problem text, provides explicit problem solving and metacognitive strategy instruction, and focuses on the flexible use of multiple solution strategies. Eighty-two teachers/classrooms with a total of 1,999 seventh-grade students across 50 school districts were randomly assigned to a treatment (SBI) or control (business-as-usual) condition. An observational measure provided evidence that the SBI intervention was implemented with fidelity. Results of multilevel modeling indicated that the SBI group scored on average significantly higher than the control group on the posttest and retention test (9 weeks later) and also showed significantly more growth in proportional problem solving. There were no treatment effects on the Process and Applications subtest of the Group Mathematics Assessment and Diagnostic Evaluation. These results demonstrate that SBI can be more effective than the control approach in improving students' proportional problem solving.

KEYWORDS: schema-based instruction, seventh-grade students, proportional problem solving

Effects of a Research-Based Intervention to Improve Seventh-Grade Students' Proportional Problem Solving: A Cluster Randomized Trial

There is considerable evidence that as children progress from the elementary grades into middle school, the mathematical gains made in elementary school are not matched in later years (e.g., National Mathematics Advisory Panel [NMAP], 2008). For example, on the 2013 National Assessment of Educational Progress (NAEP), only 35% of U.S. eighth-grade students compared to 42% of fourth-grade students were proficient or advanced in their knowledge of mathematics (National Center for Education Statistics, 2013a). Although U.S. students' mathematics scores on international assessments have improved over the past decade, data from the Trends in International Mathematics and Science Study (TIMSS) showed no measureable difference in average U.S. mathematics scores at grade 8 in 2007 and in 2011 (Provasnik, Kastberg, Ferraro, Lemanski, Roey, & Jenkins, 2012). Similarly, there were no significant changes in the average performance of U.S. 15 year olds in mathematics between 2003 and 2012 on the Program for International Student Assessment (PISA; Kelly, Xie, Nord, Jenkins, Chan, & Kastberg, 2013). Despite having some of the highest per-pupil expenditures in the world, the percentage of U.S. top performers was well below average in mathematics on the PISA, which includes tasks that require well-developed thinking and reasoning skills. These findings are cause for concern given an increasingly competitive job market, where the demand for mathematics intensive science and engineering jobs are outpacing overall job growth three-to-one (NMAP, 2008). Clearly, the need for identifying interventions that improve students' mathematics skills is critical.

The goal of the present study was to test a research-based intervention, schema-based instruction (SBI), designed to improve students' proportional reasoning. Although the SBI intervention was developed and used extensively to solve arithmetic word problems, recent work on SBI has focused on students' development of proportional reasoning. Proportional reasoning is of primary importance during the upper elementary and middle school grades (NMAP, 2008), and is one of four critical mathematics topics at Grade 7 in the Common Core State Standards (CCSS; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). In this introduction, we describe the importance of proportional reasoning and introduce the theoretical framework for SBI. Then, we summarize prior research evaluating the SBI intervention on problem solving, especially proportional reasoning, and close with a discussion of how the present study extends the literature.

Proportional Reasoning

Proportional reasoning, which is fundamental to the productive growth of mathematical reasoning, is considered key to progress in more advanced mathematics, including algebra (Boyer, Levine, & Huttenlocher, 2008; NMAP, 2008). Proportional reasoning refers to the understanding of

> structural relationships among four quantities (say a, b, c, d) in a context simultaneously involving covariance of quantities and invariance of ratios or products; this would consist of the ability to discern a multiplicative relationship between two quantities as well as the ability to extend the same relationship to other pairs of quantities (Lamon, 2007, p. 638).

Many children and adolescents, as well as adults, are not proficient with fractions, ratios, and proportions, which are essential concepts of proportional reasoning (Adjiage & Pluvinage, 2007; Fujimura, 2001; Lamon, 2007; Lobato, Ellis, Charles, & Zbiek, 2010; Miyakawa & Winslow, 2009; NMAP, 2008; Tourniaire & Pulos, 1985). Much of the literature in mathematics education indicates that young students' difficulty with proportional reasoning is related to their development of multiplicative versus additive reasoning (Lamon, 1993, 1995; Thompson & Thompson, 1994). Students often employ strategies (e.g., buildup or double counting process strategy; calculating the value of one unit and then multiplying by that value to get the desired amount; partitioning) without understanding the multiplicative nature of proportions or the composite nature of ratios. Alternatively, they tend to use many erroneous strategies in solving proportional reasoning tasks (e.g., focusing on nonmathematical reasons when comparing two ratios, "ignoring part of the data in the problem, looking at ratios of differences between the same variables" [the additive strategy]; Ozgun-Koca & Altay, 2009, p. 31).

Proportional reasoning is a complex concept that not only requires understanding the concept of ratios and that two or more ratios are equal but also requires the ability to extract relevant information to develop a representation of the problem situation (Al-Wattban, 2001). Often, the topics of ratio and proportion are presented in the context of word problems. Solving even simple proportion problems is challenging for students when they lack understanding of the problem situation and whether a solution strategy is applicable (Weinberg, 2002). Not surprisingly, many students require instruction that supports the development of underlying concepts and flexible procedures to solve proportion problems (NMAP, 2008; Tourniaire & Pulos, 1985).

Over time and with focused instruction in various linear functions in mathematics and science, secondary school students may learn to reason proportionally. However, researchers have documented another problem that tends to show up – students apply the notion of linearity to most situations even when it is not applicable (Fernández, Llinares, Van Dooren, De Bock, & Verschaffel, 2012; Van Dooren, De Bock, Evers, & Verschaffel, 2009; Van Dooren, De Bock, Hessels, Janssens, & Verschaffel, 2005). It is this overreliance on linear methods, also referred to as the illusion of linearity, that affects students' reasoning and problem solving in that they typically do not distinguish proportional from non-proportional situations (e.g., Van Dooren et al., 2005). The overuse of proportionality is often seen in classrooms that employ missing-value proportional reasoning tasks and where the focus is on technically correct procedures (Van Dooren, De Bock, Vleugels, & Verschaffel, 2010). As such, students tend to associate missing value problems with proportional reasoning and solve them using algorithmic procedures (i.e., cross multiplication) without understanding why they are applicable (Van Dooren et al., 2010). Although student difficulties with proportional thinking may be explained in part by 'routine expertise' versus 'adaptive expertise' (see Hatano, 2003), the overuse of linearity may indicate a lack of attention to the initial components of mathematical modeling – understanding the problem and the relationship between the relevant elements in the problem to translate into a mathematical model (see Verschaffel, Greer, & De Corte, 2000).

Although research on ratios and proportional relationships was prominent in the 1980s and early 1990s (e.g., Behr, Wachsmuth, Post, & Lesh, 1984; Harel, Behr, Post, & Lesh, 1992; Carpenter, Fennema, & Romberg, 1993), and scholars in mathematics education continued to explore ways to improve students' learning of this important topic, previous intervention research has been limited in several ways. First, most interventions (e.g., inquiry methods that encourage students to construct knowledge of proportionality through collaborative problem solving activities, use of pictorial representations or manipulative models) were short-term and did not focus on the broad domain of ratios and proportional relationships (Adjiage & Pluvinage, 2007; Fujimura, 2001; Myakawa & Winslow, 2009). Second, many of the studies used quasi-experimental research designs or a teaching experiment (including lesson study, action research) to evaluate the effectiveness of different approaches to the teaching and learning of proportional reasoning, limiting causal inferences. Third, these studies typically involved a limited number of students or classrooms (Adjiage & Pluvinage, 2007; Fujimura, 2001; Myakawa & Winslow, 2009). Fourth, most of this research was conducted outside of the United States with non-English speaking students (e.g., Adjiage & Pluvinage, 2007; Fujimura, 2001; Myakawa & Winslow, 2009; Ozgun-Koca & Altay, 2009); thus, the results of these studies may not generalize to middle school students in the United States.

With the exception of a few studies that have tested the effectiveness of a comprehensive curriculum package (e.g., Connected Mathematics Project; see Ben-Chaim, Fitzgerald, Benedetto, & Miller, 1998), previous research using randomized studies with teachers in various settings implementing the intervention with fidelity is sparse. The present study addressed these limitations by using a randomized design with a relatively large sample of teachers to investigate the effects of the SBI intervention, which incorporates key instructional practices identified in policy reports and research articles (see Theoretical Framework section). Further, the content in the current SBI intervention covered the CCSS content standards of ratios and proportional relationships. Specifically, the focus was on proportional problem solving involving ratios/rates and percents with the intervention helping students make sense of their reasoning related to proportions.

Theoretical Framework

SBI's theoretical framework is an elaboration of schema theory and is guided by cognitive models of mathematical problem solving (Mayer, 1999). Our SBI intervention integrates four major components; it includes a focus on the mathematical structure of problems, use of visual representations, explicit problem solving and metacognitive strategy instruction, and an emphasis on procedural flexibility. These components correspond with the recommendations articulated in the What Works Clearinghouse's recent research synthesis on improving students' mathematical problem solving performance (Woodward et al., 2012) and address the mathematical practices (e.g., look for and make use of structure, model with mathematics) in the CCSS.

Focusing on the mathematical structure of proportion problems. Prior research has found that providing students with problem categories and ways to place problems within categories improves problem solving substantially (Chen, 1999; Quilici & Mayer, 1996, 2002). Historically, there is strong evidence of the benefits of arithmetic word problem solving instruction that teaches students to identify problems of a given type (i.e., change, combine, compare) by focusing on the problem structure (e.g., Fuchs et al., 2008; Fuchs et al., 2010; Fuson & Willis, 1989; Jitendra et al., 2007). In the current SBI intervention, we focused on a less well-established typology of basic problem types on mathematical subtopics of ratio, proportion, percent/percent of change within the broad domain of proportion. Recent research suggests that focused instruction on identifying the problem structure of proportion problems is beneficial (see Jitendra et al., 2009; Jitendra, Star, Dupuis, & Rodriguez, 2013; Jitendra, Star, Rodriguez, Lindell, & Someki, 2011).

Using visual representations as mathematical tools. There is a growing body of evidence that teaching students to model problems using representations (e.g., diagrams) to make visible the underlying problem structure has a positive effect on students' problem solving performance (e.g., Fuchs et al., 2008; Fuson & Willis, 1989; Jitendra et al., 2007, 2009, 2011, 2013; Xin et al., 2011). It is important to teach students a few types of visual representations (e.g., tables, graphs, diagrams) that effectively link the relationships between the relevant quantities in the problem with the requisite mathematical operations needed to solve the problem and provide instruction on how to represent the problem using a visual representation (Woodward et al., 2012). Our SBI intervention uses schematic diagrams that help students to represent the mathematical information in problems.

Addressing problem solving and metacognitive strategy instruction. Research suggests that effective instructional practices (e.g., “explicit teacher modeling and instruction, guided questions, and efforts to engage students in conversations about their thinking and problem solving,” Woodward et al., 2012, p. 7) to represent, analyze, and solve problems serve to enrich the learning of mathematical concepts and notations and can have a direct effect on students’ achievement. Furthermore, there is compelling evidence that metacognitive strategy instruction, such as monitoring and reflecting during problem solving enhances students’ mathematical reasoning (Kramarski & Mevarech, 2003; Mevarech & Kramarski, 2003). Such instruction helps “students think about what they are doing and why they are doing it, evaluate the steps they are taking to solve the problem, and connect new concepts to what they already know” (Woodward et al., 2012, p. 17). We included metacognitive instruction as a component of problem solving instruction in the SBI intervention to teach students not only how to solve a problem, but also how to monitor and reflect on their problem-solving processes.

Emphasizing procedural flexibility. Research supports the positive effects of teaching and encouraging problem solvers to use a variety of strategies for solving problems (see Woodward et al., 2012). Understanding when, how, and why to use a broad range of methods for a given class of problems improves procedural knowledge (Star, 2005, 2007). Furthermore, there is compelling evidence that instruction that supports using, sharing, comparing, and contrasting multiple solution methods can improve students’ procedural flexibility (e.g., Star & Rittle-

Johnson, 2008, 2009). In the current intervention, the emphasis was on solving problems in different ways using appropriate methods.

Evaluation of SBI Intervention on Proportional Problem Solving

Only a few studies have conducted causal studies of the effects of the SBI intervention on solving proportion problems involving ratios/rates and percents (Jitendra et al., 2009, 2011, 2013; Xin, Jitendra, Deatline-Buchman, 2005). Xin et al. developed and tested the effectiveness of the SBI intervention with middle school students struggling in mathematics. The intervention focused on teaching a limited set of topics – proportion and multiplicative compare word problems – in 12 one-hour small group tutoring sessions. Using random assignment, scores on researcher-developed measures showed that students in the SBI group on average outperformed students in the control condition by d = 1.69 *SD*. The positive effects on proportional problem solving attributed to SBI strengthened on a retention test (d = 2.53).

Using a more comprehensive coverage of topics that included ratios, equivalent ratios, ratio word problems, rates, proportion word problems, scale drawing problems, Jitendra et al. (2009) studied the effectiveness of the SBI intervention with seventh-grade students. Eight seventh-grade classrooms were randomly assigned to SBI or a "business as usual" control condition. Students in both conditions were instructed five times a week for 45 min over a 2-week period. Compared to the control condition, students in the SBI group scored on average significantly higher on proportional problem solving (d = 0.45) and maintained the effects on a 4-month retention test (d = 0.56).

To address limitations of the Jitendra et al. (2009) study, a follow-up study was designed that targeted additional topics such as proportional problem solving involving percents, included more classrooms (j = 21) across three schools in two suburban school districts, extended instructional time over a 6-week period, and provided longer professional development to classroom teachers (Jitendra et al., 2011). Seventh-grade classrooms were randomly assigned to SBI or control conditions, and results indicated that the posttest difference favoring the SBI group was statistically significant for proportional problem solving (multilevel standardized effect size = 0.32). However, the effects of SBI were not maintained on the retention test given a month after the end of the intervention. The authors attributed this finding to a lack of power to detect significant differences given the modest number of classrooms ($j = 21$).

Using a randomized design, Jitendra et al. (2013) conducted a rigorous replication of Jitendra et al. (2011) that increased the sample size to include more classrooms ($j = 42$) across more schools ($k = 6$) and reduced direct involvement of the research team. Fidelity of implementation was assessed with videotaped sessions of both SBI and control classrooms. With these methodological improvements, students in SBI classrooms on average outperformed students in control classrooms on a measure of proportion problem solving at posttest (multilevel standardized effect size = 0.36) and maintained their problem solving skills at 6 weeks follow-up (multilevel standardized effect size = 0.29).

The Present Study

While previous studies have supported the efficacy of SBI, each of the evaluations involved small to modest numbers of teachers (range = 6 to 42) and their students for a geographically limited sample of schools (range = 1 to 6) that used a total of four different mathematics programs. The research design in these studies consisted of randomly assigning classrooms to SBI intervention or control (business as usual) conditions. Teachers with multiple classrooms taught both SBI and control classrooms all of which were included in the study. In the current study, we were interested in determining whether a similar implementation of SBI, as in Jitendra et al. (2013), would have comparable effects for a sample of greater geographic and socioeconomic diversity of schools and students, and a larger number of teachers. Another important feature of the study was eliminating the direct involvement of the research team in supporting both SBI classroom implementation and test administration. Research team members supported teachers primarily on logistical issues rather than the level of curriculum implementation intensity provided in previous studies.

Additionally, we improved our study design by randomly assigning teachers to SBI or control and then randomly selecting one of their classrooms to participate in the study, meaning that each teacher in the current study taught in a SBI or control classroom but not both. By implementing SBI in 50 school districts across a state in the upper Midwest reflecting urban, suburban, and rural school settings we also assessed whether the efficacy of SBI would hold when control classrooms used increasingly diverse mathematics programs. Another purpose of this study was to document the effects of SBI on proportional problem solving related to student background variables like socioeconomic status or sex.

We had three specific research questions. Our first research question examined whether SBI leads to improved proportional problem solving performance compared to a business-as-usual instruction control group and whether students' proportional problem solving skills would be maintained 9 weeks after the termination of the intervention. Based on previous research, we hypothesized that SBI would on average increase student understanding of proportional reasoning compared to a control group (e.g., Jitendra et al., 2009, 2011, 2013).

The second research question examined whether SBI results in increased achievement compared to a control group on overall mathematical problem solving performance after a focused period of time spent on ratios and proportional relationships. We used a norm-referenced standardized test that measures application of mathematics concepts in multiple content areas (e.g., number, data analysis, geometry) to examine overall problem solving achievement and what students might be expected to know during the entire year. Given that 67% of the items on this test are based on concepts not taught in the study, we hypothesized that students' overall problem-solving achievement in the two groups would be comparable.

The third research question we tested was whether SBI and teacher-classroom characteristics moderate the effects of student-level background variables (e.g., socioeconomic status, sex) on mathematical problem solving. We hypothesized that treatment classrooms would be associated with weaker relationships between background variables and understanding proportional reasoning, compared to control classrooms, which would provide evidence of the moderating effect of SBI.

Method

Setting and Sample

Middle school math teachers in an upper Midwest state who taught a typical seventh-grade math class (approximately 600) were invited to participate in the study. Of those, a total of 82 seventh-grade math teachers from 58 middle schools across 50 districts volunteered to participate in the study. Student enrollment in the 50 districts ranged from 115 to 37,864 students, with an average enrollment of 1,698 students. Of the 50 districts, 38 (76%) were located in rural settings, eight (16%) in suburban settings, and four (8%) in urban settings. The percent of minority students in the districts ranged from 0 to 76% with an average of 34%; the percent of students eligible for free or reduced price lunch in districts ranged from 15 to 73% with an average of 52%. Student and teacher participant information is provided in the following sections.

Students. The sample consisted of 1,999 seventh-grade students; the majority of students were White (77%), with 9% Black, 7% Hispanic, 6% Asian, and 1% American Indian. Due to data analysis difficulties linked to small sample sizes and the presence of missing data, the $n = 18$ American Indian students were removed; as such, the student sample used for the present analyses consisted of 1,981 students (fewer in some analyses because of missing data). The mean age of these students was 12 years, 8 months (*SD* = 4 months). Approximately 40% of the total sample of students was eligible for a free or reduced price lunch, 10% received special education services, and 6% were English language learners (see Table 1 for student demographic information).

Teachers. The 82 participating seventh-grade mathematics teachers' mean years of experience teaching mathematics was 11.9 (*SD* = 6.4, range 1 to 34 years). All teachers were certified to teach mathematics; 6% were also certified to teach science, 15% were certified in subjects other than mathematics or science, and 36% were certified in all subjects (generalist). Similarly, all teachers were certified to teach grades 6-8; in addition, 55% were also certified to teach grades 9-12 and 45% to teach grades K-5. Virtually all of the teachers were White ($j = 79$), with one Hispanic, Asian, and American Indian teacher. Sixty-seven percent of teachers were female (see Table 1 for teacher demographic information). Forty six (56.1%) teachers taught in schools classified as located in a rural setting, 27 (32.9%) in a suburban setting, and 9 (11%) in an urban setting.

Study Design

We used a prospective randomized cluster design with longitudinal (pretest, posttest, delayed posttest) data in which teachers/classrooms served as clusters. If properly implemented, this design ensures that estimated treatment effects are unbiased (Bloom, Richburg-Hayes, &

Black, 2007). Initially, one class of students for each of the 82 teachers was randomly selected to participate in the study. Then, each of the 82 teachers and their participating class (cluster) was randomly assigned to one of two conditions: treatment (j = 40) or control (j = 42). The unequal number of teachers/classrooms was due to one teacher originally assigned to the treatment condition being subsequently moved to the control condition for logistical reasons. Immediately following random assignment one teacher indicated that she could not attend PD, which precluded her from being a treatment teacher. The teacher was moved to the control condition before the PD training. As such, the change in assignment could not have affected the teacher's fidelity implementation, which was more similar to the control condition.

An a priori power and sample size analysis using the Optimal Design software (Spybrook et al., 2011) was performed that focused on testing the SBI vs. control effect for cross-sectional data needed to answer the research questions. The results indicated that 82 clusters and 1,900 students would allow us to detect a standardized effect of .30 (a moderately small effect following Cohen, 1988) for the SBI vs. control comparison, with a power of .80 for an intra-class-correlation of .19 taken from Hedges and Hedberg (2007) for seventh grade mathematics data, and a power of .95 under the same conditions for a standardized effect of .40.

Treatment Instruction

Treatment teachers participated in 16 hours of professional development (see Professional Development section) in mid-December, followed by delivering the SBI intervention five days a week across six weeks between January and February. Teachers replaced the lessons on ratio/proportion and percent in their curriculum with the SBI lessons.

SBI intervention. The SBI program content consisted of two replacement units, one focusing on Ratio/Proportion and the other on Percent. Each unit comprised ten 50-min lessons with an additional lesson at the end of the second unit that provided practice on solving problems related to ratio, proportion, and percent. The SBI program was aligned with the state mathematics standards and covered the same content taught in seventh-grade classrooms (for further details about the content of SBI see Jitendra et al., 2011). SBI teachers were provided with (a) a detailed teacher guide that we developed as a resource to fully understand program features, along with teaching materials (e.g., visual diagrams and problem solving checklists) and student materials (i.e., workbook and homework book) to support implementation of activities to develop critical concepts and skills, and (b) professional development to help teachers use the lessons. The SBI intervention and professional development are described in greater detail next.

SBI's instructional approach was designed to include four instructional practices: (1) explicitly modeling problem solving and metacognitive strategies, (2) activating the mathematical structure of problems, (3) visually mapping information in the problem using schematic diagrams (see Figure 1), and (4) developing procedural flexibility. With these four practices, teachers initially modeled problem solving by thinking aloud (see Appendix A, sample excerpt of the script for solving a problem in Lesson 11 located in the online supplemental materials) and gradually shifted responsibility to the students by scaffolding instruction using teacher-student dialogues to help clarify and refine their thinking (see Appendix A, sample excerpt of the script for percent of change problem solving in Lesson 14 located in the online supplemental materials).

The aim of the first practice was to promote mathematical problem solving and metacognition on the basis of extensive modeling and scaffolding by the teachers. Using a four-step problem solving strategy, represented by the acronym DISC (Discover the problem type, Identify information in the problem to represent in a diagram, Solve the problem, Check the solution), teachers first focused on activating the mathematical structure of the problem (second practice). Through classroom discussions and by answering deep-level questions, teachers encouraged students to identify the type of problem (i.e., ratio, proportion, or percent) by reading, retelling, and examining information in the problem as well as thinking about how problems within and across types are similar or different. For example, students learned that ratio and proportion problems are similar because they both involve a multiplicative comparison of two quantities and are different in that ratio problems are confined to a single situation, whereas proportion problems describe a statement of equality between two ratios/rates that allows one to think about the ways that the two situations are the same.

Second, students learned to connect the problem to a certain schematic diagram and used the appropriate diagram to represent the problem such that the visual mapping showed the relevant elements, relations and conditions embedded in the problem (third practice). Specifically, instruction focused on identifying information critical to solving the problem to represent using the schematic diagram. Through careful instruction, students also reasoned why the same ratio schematic diagram can be used to represent information in both ratio and percent problems (a percent is a special type of ratio). With further instruction, students understand that while ratio diagrams work well for some percent of change problems to represent the relation between the change amount and original amount, more complex percent of change problems (including simple interest) require the use of diagrams that depict both multiplicative and additive relationships (see Percent of Change diagram in Figure 1).

Third, students estimated an answer, made decisions about what method (equivalent fractions, unit rate, cross multiplication) to use to solve the problem (fourth practice), and solved the problem. With explicit instruction on multiple solution methods for solving proportion problems, the goal of such instruction was to have students become cognizant of specific methods that are more efficient than others and select the strategy that is most efficient based on the numbers in the problem.

Finally, teachers encouraged students to use their estimated answer from the previous step in evaluating their work to determine whether the answer made sense. At each stage in the problem-solving process, teachers used prompts or deep-level questions to encourage students to monitor and reflect while solving a problem. For example, prompts were used to ensure that students (a) understand and identify the problem type (e.g., Why is this a *proportion* problem? How is this problem similar to or different from one I already solved?), (b) identify and represent the critical information in the problem using an appropriate diagram (e.g., Which diagram is best to represent information in the problem?), (c) select a strategy to solve the problem (e.g., Which solution method would best help me solve this problem?), and (d) check the solution (e.g., Is the answer reasonable based on my estimate?).

Professional development. The goal of the two-day professional development provided by one of the authors of the SBI program was to support teachers' learning and implementation of SBI, as well as to provide training in implementing the new approach and assessments faithfully. In the first training session, teachers were introduced to the project and key features of the SBI intervention (e.g., recognizing problem types, generating estimates, knowing multiple strategies) to support student learning of ratio, proportion and percent. Teachers practiced with SBI techniques and materials to sort problems by type of problem, use schematic diagrams to represent information in the problem, generate "ballpark" estimates (i.e., quick and easy based on benchmark numbers and fractions), and select an appropriate solution method from among several strategies to solve problems.

The second training session gave teachers an opportunity to review the two curricular units on ratios/rates and percent along with associated materials and to learn about implementing the intervention. Throughout the training, the focus was on developing students' proportional reasoning with the SBI intervention. Teachers viewed multiple short video segments of teachers from a previous study to illustrate the implementation of the intervention, with the focus on eliciting student discussion. The video segments provided an opportunity to address the importance of implementing SBI intervention faithfully without the need to read the teacher guide while teaching. In addition, the training emphasized the importance of treatment teachers not sharing project materials or strategies with any control group teachers in their building.

"Business as usual" control instruction. Students in the control condition were taught the topics of ratio, proportion, and percent using their district-adopted textbooks in the same time period as the treatment condition. We gathered information on textbooks used in the control classrooms from a written teacher questionnaire in which teachers listed the mathematics textbooks they used and a review of lessons on ratio/proportion and percent topics sampled from the textbooks. Overall, teachers in the control classrooms used traditional resources that consisted of 10 different textbooks published from 2001 to 2012 by one of three publishers: Houghton, Mifflin, Harcourt; Glencoe/McGraw Hill; Pearson Education. These textbooks were examined for the presence of the core instructional components of SBI (i.e., identifying the problem type, visual representations, modeling of problem solving and metacognitive strategies, multiple solution strategies).

Several control textbooks included instructional components such as the use of visuals, problem solving procedures, and multiple solution strategies. For example, all of the control textbooks included various visual representations (e.g., pie graphs, percent bar graphs, strip diagrams). However, none provided instructions to activate the mathematical problem structure using the visuals. Although about one-half of the control curricula also incorporated problem-solving instruction (e.g., “use a table to set up a proportion, write a proportion, multiply each side by 100, simplify;” Larson, Boswell, Kanold, & Stiff, 2007, p. 348), the emphasis was more on problem solution procedures and less on problem comprehension. Metacognition, when included in the textbooks, was not explicitly targeted for instruction (e.g., reflect using a math journal). Approximately one-third of the control textbooks encouraged the use of various solution strategies (e.g., recognizing that there is more than one way to solve a proportion problem); however, none emphasized the selection of the most efficient strategy based on the relationships between the numbers in the problem. In short, our review of control classroom texts suggested that the instructional components covered do not overlap with those in treatment classrooms in ways that would distort estimates of the effects of SBI. To assess the impact of different curricula used in control classrooms we performed analyses comparing the proportional problem-solving (PPS) and Group Mathematics Assessment and Diagnostic Evaluation (GMADE) test scores of control students as a function of the curriculum they experienced.

Measures

Proportional problem-solving (PPS) test. We assessed students’ proportional problem solving performance using the PPS test, which we developed using released items related to the topics of ratio, proportion, and percent from NAEP and TIMSS as well as questions from past state mathematics assessments (see sample items in Appendix B located in the online supplemental materials). The same test was used for pretest, posttest, and delayed posttest. The delayed posttest was given 9 weeks after the end of the intervention. The PPS test included multiple-choice items and short-response items that addressed the general program content of ratio/proportion and percent.

The 23 multiple-choice items were dichotomously scored. We scored the four short-response items on the PPS test using a rubric, which emphasized correct reasoning; responses were scored on a 0-to-2 point scale. Students' scores on the PPS test were calculated by taking the sum of their points earned (total possible points equaled 31), which means that the short-response and multiple-choice items were unequally weighted.

To score the short-response items, we developed a rubric based on a sample of student responses from a previous study using the same measure (Jitendra et al., 2013). Next, all raters participated in training that involved scoring several sample student responses until at least 90% agreement between raters was reached. The pretest, posttest, and delayed posttest short-response items were scored by one of several project staff members who were blind to the student group. To assess the degree of inter-rater consistency a different rater scored 33% of the short-response items, producing intra-class correlations 0.85, 0.91, and 0.89 at pretest, posttest, and delayed posttest, respectively.

To assess the reliability of the PPS test we followed recommended practice (Dunn, Baguley, & Brunsden, 2013) and performed separate analyses for the pretest, posttest, and delayed posttests using the jMetrik software (Meyer, 2007). jMetrik is a comprehensive item analysis software package that assist users in identifying the measurement model that best captures patterns in the item responses, and provides a range of useful statistics such as reliability coefficients. The jMetrik software identified the congeneric measurement model as providing the best fit to the PPS item responses (RMSEA values $\leq .03$; GFI values $\geq .97$), and we report the (omega) reliability coefficient associated with this model.

A congeneric measurement model assumes a single continuous latent factor underlies (in our case) the dichotomous- and trichotomously-scored PPS items but places fewer assumptions on the data than other models, such as that assumed to underlie the traditional alpha coefficient of reliability (McDonald, 1999). Specifically, in a factor-analytic framework the congeneric model allows item loadings to vary and tends to produce unbiased (or less biased) estimates of reliability compared to the traditional alpha reliability coefficient, especially if item response formats vary (McDonald, 1999) as is the case with the PPS. The omega reliability coefficient associated with the congeneric model represents a ratio of the estimated true score variance to observed score variance obtained from a factor analysis and thus has values between 0 and 1, with higher values indicating greater reliability (Dunn et al., 2014; Revelle & Zinberg, 2009). For the PPS pretest, posttest, and delayed posttest the estimated omega reliabilities were 0.69, 0.77, and 0.76, respectively. Because omega can be interpreted like the traditional alpha coefficient of reliability in that values closer to one signal less measurement error, the PPS reliabilities indicate moderate reliability (Nunnally, 1978, p. 245). We recognize that reliabilities in this range may be deemed inadequate by some researchers. However, it is important to emphasize that we found statistically significant treatment effects for the PPS posttests (see below) despite the random measurement error linked to these reliabilities. Thus, while the reliabilities for the PPS were somewhat smaller than desired they did not compromise our ability to detect treatment effects.

Group Mathematics Assessment and Diagnostic Evaluation (GMADE). We used the Process and Applications subtest of the GMADE (Pearson, 2004), Level M, Form A, a norm-referenced standardized assessment, at both the pretest and posttest to assess students' overall mathematics problem solving performance. The construction and validation of this test was informed by the Principles and Standards for School Mathematics of the National Council of Teachers of Mathematics (NCTM, 2000). The Process and Applications subtest specifically measures students' ability to comprehend mathematical language and concepts and apply relevant operations to solve word problems across multiple content areas (e.g., algebra, geometry, number and operations). This allowed us to examine student performance on a general measure of problem solving rather than only on ratio/rates and percent problem solving. The Process and Applications subtest includes 30 multiple-choice items that require students to read a short passage of one or more sentences and choose the best of four possible answers, in which choices included numbers, pictures, or symbols (Williams, 2004). Some items also included multiple-step problems and Process Problems that require identifying a process (e.g., reasoning) or application to derive the answer. A Process Problem is one that does not require students to solve the problem, but has them think about the process for solving it. For example, students would be presented with a problem and asked a process question such as, "What is the appropriate first step to solve the following problem?" (Pearson, 2004, p. 22).

All items were scored as correct or incorrect. Once again the congeneric model best fit the item data, with coefficient omega reliabilities for the pretest and posttest of 0.61 and 0.69.

The PPS and GMADE assessments were group-administered in classrooms by the classroom teacher following standardized protocols. Both tests were untimed, but each test could be completed in 50 minutes on average. All students were administered the pretests in December of the school year and posttests during the week following the last day of the intervention, with the PPS test also administered nine weeks later.

Fidelity of implementation. To document and measure fidelity of implementation, we developed two measures based on guidelines proposed by O'Donnell (2008) that addressed procedural fidelity and adherence to the SBI intervention (Dane & Schneider, 1998) as well as overall quality of instruction in treatment and control classrooms based on attributes of effective teaching. Fidelity of implementation and quality of instruction information was generated by videotaping an entire lesson on proportion problem solving for each teacher during the 6 weeks of the study. We selected proportion problem solving lessons to ensure that the observed tasks incorporated the core features that were targeted for evaluation. We assessed procedural fidelity and adherence by observing videotaped lessons using a checklist developed to document the presence of the core features of the SBI intervention. The same checklist was also used in the control condition to evaluate program differentiation and determine whether control teachers spontaneously provided instruction that was similar to the key elements of SBI (Dane & Schneider, 1998). For each lesson, raters completed seven items evaluating whether teachers completed all components corresponding to the four SBI instructional practices and whether they were fluent in facilitating student thinking. The seven items were: (a) identifies the problem type by focusing on the key problem features, (b) connects the new problem to previously solved problems, (c) represents critical information in the problem text using an appropriate diagram, (d) generates an estimate prior to solving the problem, (e) discusses multiple solution strategies, (f) solves the problem and presents the solution within the context of the problem, and (g) evaluates the solution.

Procedural fidelity items were coded on a 0-to-3 scale (3 = high level of implementation – 0 = did not implement). The coding scheme for the fidelity measure was developed by one of the authors in consultation with the first author (the SBI program developer), with the measure finalized after multiple rounds of independent video coding by six coders, discussion, and re-operationalization of the codes. During coder training a benchmark of 90% agreement on applying codes was treated as adequate. Fidelity was independently assessed for each classroom video by two coders, producing a total of 160 codings (i.e., two per classroom). We were not able to record two control classroom teachers because of scheduling conflicts. Disagreements in coding were resolved through discussion and review of the videotapes. Inter-rater reliability was estimated by computing intra-class correlations for the coder ratings and averaged 0.98 across the seven items (range 0.97 to 0.99).

The overall quality of instruction was assessed using four items that focused on features such as the teacher's ability to clarify the lesson purpose, provide lesson closure, manage instructional time (i.e., how well the teacher managed student behavior), and minimize mathematical errors. The items were evaluated on the same 0-to-3 scale as the fidelity measure, and inter-rater reliability for the coder ratings averaged 0.99 across the four items (range 0.96 to 1.00).

Data Analysis

Descriptive statistics for the measures by treatment are presented in Table 3. We also compared the scores of control students on the PPS and GMADE assessments as a function of the mathematics curriculum they experienced; nonsignificant results imply that the control curricula did not differentially affect student PPS and GMADE scores and strengthen arguments for pooling control classrooms.

To assess differences between the treatment and control classrooms we fitted a series of multilevel (i.e., two-level, students within clusters) models with covariates at both levels using the HLM 6 software (Raudenbush, Bryk, & Congdon, 2004). Adjusting the data for control variables can account for variation that otherwise remains unexplained and improve estimation and statistical power (Bloom et al., 2007). The outcome variables included in the analyses were the PPS posttest, PPS delayed posttest, and GMADE posttest, which were analyzed separately. We also performed an ancillary analysis of the PPS longitudinal data to explore student change over time and whether such change was impacted by the treatment.

For each outcome, the Level 1 (student) model contained three covariates: pretest score, sex (1 = males, 0 = females), and race (Black = 1, Hispanic = 1, and Asian = 1, White = 0 so the latter served as the reference group). All Level 1 covariates were grand-mean centered.

Level 2 variables included the treatment variable (1 = SBI, 0 = control), five teacher covariates (i.e., number of post-secondary mathematics courses taken, number of post-secondary education courses taken, years of teaching experience in mathematics, number of PD hours in mathematics or mathematics education in the last year, sex), and variables capturing the percentage of limited English proficiency (LEP), eligible for free or reduced price lunch (FRL), and students receiving special education services per classroom or teacher. Given that there was little or no variation in classrooms with regard to the sample characteristics such as LEP, FRL, and special education (e.g., one-half of the classrooms had no LEP students) and thus compromised estimation of model parameters within each classroom, we decided to aggregate these variables to the classroom level (e.g., %LEP students in a classroom). Because the distributions of the percentages were ragged and discontinuous we rescaled these variables to quintiles and used the rescaled versions as Level 2 covariates.

Slopes capturing the impact of student variables (e.g., sex) on the outcome variables were found to be statistically equal across classrooms (i.e., variance = 0). Thus models examining the impact of treatment on Level 1 relationships were not fitted, and in what follows the results are based on intercepts-only models.

We examined the data for evidence that model assumptions were satisfied for all analyses, and no major violations were found for any of the models. To control for compounding of Type I error rates several methods are available. We used the Dunn-Bonferroni correction (Miller, 1966) in which an overall (i.e., experimentwise) Type I error rate (e.g., $\alpha = .15$) is divided among all statistical tests linked to each outcome variable with no requirement that the error rate be divided equally. Accordingly, we assigned .05 to the test of the treatment effect because this was the most important effect in the model and divided the remaining .10 among tests of the remaining fixed effects, producing $\alpha' = \frac{.10}{14} = .0071$. Also, for both descriptive and inferential analyses we used all available data, which means that sample sizes vary across analyses depending on attrition patterns (missing data).

Attrition analysis. An examination of missing data indicated that it centered on student variables and that for all outcomes, except the PPS delayed posttest, attrition was less than 5%. Importantly, attrition was approximately equal in the treatment and control groups. For the PPS delayed posttest, 6.8% of students had missing data because the test was not administered in two control classrooms due to end of the school year time constraints. The percentages of missing data for all student-level covariates were below 5%.

Small percentages of missing data are unlikely to bias findings if the percentages are similar across groups (Peng, Harwell, Liou, & Ehman, 2006), but as a sensitivity analysis we also fitted each of the final HLM models to the student sample that provided complete data (no missing data) and compared the results to those obtained using all available data (some missing data). The results of these models did not differ in any significant way, suggesting that our models were insensitive to the presence of missing data. As such, we used all available data for each analysis meaning that student sample sizes for each analysis varied.

Results

Descriptive Results

We initially performed a series of descriptive analyses that included examining the correlations between all measures and exploring pre-existing differences between the SBI and control group students. Results of the correlational analyses showed that the correlations between the PPS pretest and posttest, pretest and delayed posttest, and posttest and delayed posttest were 0.65, 0.64, and 0.75, respectively. For example, the PPS pretest/posttest correlation of 0.65 means that students who scored above (or below) the pretest mean also tended to score above (below) the posttest mean; equivalently, $(.65)^2 = .422$ means that 42.2% of the variation in the posttest can be predicted from variation in the pretest. The correlation between the GMADE pretest and posttest was 0.54. Correlations between the PPS and the GMADE tests ranged from 0.51 to 0.61 across time points.

We also computed descriptive statistics to check whether the SBI intervention was on average implemented with fidelity. Table 2 displays both fidelity and quality of instruction data. We conducted *t*-tests to test group differences on both the fidelity and quality of instruction data and used the Dunn-Bonferroni correction to control for compounding of Type 1 error. With regard to the fidelity of implementation, the mean total score across the seven items on the fidelity checklist was 14.33 (*SD* = 3.86) for treatment teachers and 7.43 (*SD* = 3.00) for control teachers out of a possible 21 points (higher scores are consistent with greater fidelity). Results indicated statistically significant and fairly substantial differences between the treatment and control groups on the total score and all individual items except for item 6 (i.e., solves the problem and presents the solution within the context of the problem), with treatment teachers implementing SBI elements more than control teachers. For example, the effect size of 1.34 for

"Identifies Problem Type" means that on average SBI classrooms were rated 1.34 *SD* above control classrooms, or, equivalently, 91% of SBI classrooms were rated higher than the average control classroom rating on this item (Lipsey et al, 2012).

With regard to quality of instruction, we expected to observe general teacher behaviors (e.g., clarifies lesson purpose, minimizes mathematical errors) in both treatment and control classrooms. The results indicate that, on average, both treatment (M = 9.50) and control (M = 9.28) teachers were rated similarly on these behaviors, $t(78) = 0.66$, $p = .507$, indicating that on average there were no differences between the groups in terms of quality of instruction. These data (fidelity and quality of instruction) allowed us to investigate program differentiation (Dane & Schneider, 1998) in that there were clear differences in SBI instructional elements across the two groups, whereas the general quality of instruction was similar in both conditions.

Table 3 reports means and SDs for the treatment and control groups for each measure. Differences between the treatment and control groups on the GMADE pretest were not statistically significant, whereas the difference on the PPS pretest was d = -0.10 *SD*, which would be statistically significant at $\alpha = .05$ given the large sample of students. It is not clear what the source of this difference is but the inclusion of this variable as a covariate in the multilevel analyses means the outcomes will be adjusted for this difference.

Last, we fitted two-level (students-within-classrooms) multilevel models using only control classrooms with curriculum dummy-coded to the PPS total, ratio/proportion, and percent posttest and delayed posttest outcomes, along with the GMADE outcome. None of these results was statistically significant implying that for the outcomes we studied the particular curriculum used in a control classroom produced a similar impact, providing empirical evidence for pooling control classrooms.

Treatment Effects on Proportional Problem Solving

The first research question asked whether SBI leads to improved proportional problem solving performance compared to business-as-usual instruction, and whether students' proportional problem solving skills would be maintained 9 weeks after the termination of the intervention

PPS posttests. To estimate the intraclass correlation (ICC) we fitted unconditional two-level (students within teachers/classrooms) models separately to the PPS posttest and delayed posttest. The ICC was .21 ($p < .001$) for the posttest and .18 ($p < .001$) for the delayed posttest, indicating that 21% and 18% of the variance in these tests was between classrooms. These values are consistent with those for mathematics data reported by Hedges and Hedberg (2007) as typical in education. Next we fitted a model with student background variables and the PPS pretest at Level 1, and teacher covariates plus the treatment at Level 2.

The results for the PPS posttest indicated that treatment was a statistically significant predictor of PPS posttest scores, with SBI classrooms outperforming control classrooms on average, $\gamma_{01} = 1.70$, $t(72) = 5.38$, $p < .001$. The standardized effect size for the treatment effect was 0.46 *SD*, meaning that approximately 68% of treatment classrooms scored above the mean of control classrooms (Lipsey et al., 2012); alternatively, the proportion of classroom intercept variance explained by the addition of the treatment variable above and beyond that attributable to other predictors in the model was 35%.

In addition, the Black and pretest variables were statistically significant predictors of PPS posttest. The results indicated that Black students scored on average lower than White students, $\gamma_{30} = -1.06$, $t(1799) = -3.23$, $p = .002$. For the PPS pretest the results indicated that this variable was a significant predictor of the posttest, $\gamma_{80} = 0.68$, $t(1799) = 27.64$, $p < .001$ (see Table 4).

The results for the PPS delayed posttest indicated that the treatment variable was a statistically significant predictor of delayed posttest scores, with SBI classrooms outperforming control classrooms, $\gamma_{01} = 1.22$, $t(70) = 4.17$ ($p < .001$), with a standardized effect = 0.32 *SD* indicating that approximately 62% of treatment classrooms scored above the mean for control classrooms. Treatment also accounted for 32% of the between-classroom intercept variance above and beyond that attributable to other predictors in the model. The full set of HLM results for the PPS delayed posttest can be found in Table 5.

Ancillary analysis of PPS longitudinal data. We also explored the impact of SBI on students' growth using the PPS data by fitting a three-level (repeated measures within students within classrooms) model. The average PPS score at pretest was 14.0 ($p < .001$) and the average linear slope over time was 0.78 ($p < .001$), the latter indicating that student scores on average increased over time. The results indicated that the treatment variable was a statistically significant predictor of student linear growth, with SBI students having steeper learning trajectories than control students, $\hat{\gamma}_{101} = 0.63$, $t(72) = 4.86$, $p < .001$, suggesting that SBI was associated with greater growth in learning over time than the control condition.

Treatment Effect on Mathematical Problem Solving (GMADE)

The second research question examined whether SBI results in improved overall mathematical problem solving performance compared to business-as-usual instruction. The ICC for the GMADE posttest was .20 ($p < .001$), indicating that 20% of the variance in the posttest was between classrooms. Results of the fitted model for the GMADE posttest indicated that treatment was not a significant predictor of the GMADE posttest, and that the Hispanic and pretest variables were the only statistically significant Level 1 predictors. In addition, the results indicated that teachers' years of experience was a significant predictor of the GMADE posttest, $\gamma_{02} = 0.07$, $t(72) = 3.37$, $p = .002$ (see Table 6).

Moderating Effects of SBI

The third research question examined whether SBI and teacher-classroom characteristics moderated the effect of student-level background variables on mathematical problem solving. To address this research question, we fitted a model to each outcome where the Level 1 slopes associated with the student demographic variables (e.g., sex) were allowed to vary. The results of these analyses indicated no statistically significant between- classroom variance in Level 1 slopes for any of the outcomes (all $p > .01$), indicating that SBI and teacher-classroom characteristics did not moderate the effect of student-level background variables on mathematical problem solving.

Discussion

The main purpose of this study was to measure the efficacy of the SBI intervention. SBI represents a promising approach for improving students' problem solving performance based on previous research (Jitendra et al., 2009, 2011, 2013), and was extended in the current study to a larger number of teachers and students in middle schools that included rural, suburban, and urban locations, and to a setting in which the level of support to teachers implementing the intervention was removed. The first research question examined the effect of SBI on the proportional problem solving performance of seventh-grade students, and whether the effect of SBI was maintained 9 weeks after its termination. Compared to the control condition, the SBI intervention had a significant positive effect on the PPS posttest and PPS delayed posttest (i.e., the proportion of between-classroom intercept variance attributable to the PPS posttest and delayed posttest effects were 35% and 32%).

Results of our longitudinal analysis showed that students in the treatment group demonstrated significantly more growth on the PPS relative to students in the control classrooms. These positive findings are notable because the control condition covered approximately the same instructional topics as SBI and thus had the same advantage on the PPS assessment, which assessed proportional reasoning.

The effects of the SBI intervention are not only statistically significant, but also are substantively important (see What Works Clearinghouse, 2014). These results confirm findings from previous evaluation studies of the SBI intervention (Jitendra et al., 2009, 2011, 2013) and are noteworthy when we consider that the present study design was more rigorous (i.e., random assignment of teachers to the treatment and control groups), involved a greater number of teachers and students in more diverse locations, and that research staff were not present in classrooms to provide instructional support to teachers, as in previous studies. Our results also provide support for the efficacy of the SBI intervention in not only improving student learning with regard to proportional problem solving, but also the retention of the effect nine weeks after the end of the intervention, similar to prior findings (Jitendra et al., 2009, 2011, 2013).

The stronger results for the SBI condition may be explained by program differentiation results from the fidelity data, which highlighted important instructional differences between the treatment and control conditions. Although the content focus was the same in both conditions, there were some clear and meaningful differences in instructional emphasis across conditions. Treatment teachers implemented SBI lesson elements significantly more (i.e., moderate to high level of implementation) than control teachers (i.e., low level of implementation). The fidelity data indicated that the SBI lesson elements control teachers were most likely to implement represented instructional practices that most mathematics teachers typically engage in (i.e., providing a solution within the problem context). In contrast, control teachers were less likely to implement SBI lesson elements that are relatively unique to SBI (e.g., identifying the problem type).

In short, SBI, with its emphasis on the underlying problem structure that requires students to categorize problem types by discerning the relevant elements, relations, and conditions embedded in the problem, use of visual representations, and instructional strategies (problem solving, metacognition, multiple solution) that encouraged students to engage in problem solving, reason at high levels, and explain their thinking, resulted in superior problem solving performance for treatment students compared to the control group. Integrating these strategies with mathematics content is important in connecting mathematics practices to mathematical content articulated in the CCSS (2010).

The second research question examined the effect of SBI on the overall mathematical problem-solving performance of seventh-grade students. Our results indicate that the scores of students in the treatment group on a standardized test (Process and Applications subtest of the GMADE) were not significantly different from those of students in the control group. One explanation for this finding is that only 33% of the items on the GMADE focused on proportion and percent problem solving. The result supports our hypothesis and is not surprising in that the SBI intervention addressed only one of several strands of mathematics that the GMADE evaluates. However, the finding is discouraging even though it is consistent with that reported in Jitendra et al. (2011, 2013), in which the SBI intervention did not result in a significant change in performance on a domain-general measure of problem solving. Although SBI is designed to develop student competence in problem solving and proportional reasoning, and we provided multiple examples that emphasized the critical features of problem types that Wagner's (2006) theory of transfer-in-pieces argues are essential for a transfer effect, the standardized test comprised content that was less familiar and therefore less sensitive to the effects of SBI.

Taken together, these results suggest that there was value added to students' proportional problem solving performance as a result of the SBI intervention. Students in the treatment group exhibited a deeper level of reasoning and thinking on the PPS test while doing as well as students in the control group on a mathematical problem solving assessment that covers all content areas.

The third research question was whether SBI and teacher-classroom characteristics moderated the effects of student-level background variables on mathematical problem solving. Our analysis showed that student-level relationships did not vary across classrooms for the outcome variables, meaning that the moderating effect of treatment could not be studied. Perhaps the lack of variation was related to the percentage of classrooms (76%) located in rural settings and that 77% of our student sample consisted of White students. The predominance of White students in rural schools is widely documented (NCES, 2013b) and may produce a homogeneity that leads to a lack of variation in student-level relationships across classrooms. Including location of a school district as a predictor did not produce significant results or influence model results in any noticeable way.

Limitations

This study has some limitations to be considered. One limitation of the study is that control group teachers did not receive professional development. As such, it is possible that the treatment effects were due, in part, to the 16 hours of professional development training the treatment teachers received prior to the implementation of SBI. However, it is worth noting that the aim of the study was to contrast SBI instruction with typical mathematics instruction (i.e., "business as usual").

Another potential limitation in this study is that fidelity was addressed by evaluating one videotaped lesson, which may not have been representative of teachers' fidelity of implementation across the entire study. At the same time, one video-recorded observation may be sufficient, given the relatively brief period of the intervention (six weeks), to provide a representative sample of participant functioning (Breitenstein et al., 2010). Video-recorded data, which have several important advantages (e.g., ability to capture complex interactions, allow multiple viewings), helped us maintain the quality of the coding that was done. In addition, because of scheduling constraints teachers were not evaluated on the exact same lesson (although all lessons were about proportion problem-solving), which could affect the results; however, the fidelity measure was designed to be used with all SBI lessons and was implemented in both treatment and control classrooms. Finally, a novelty effect is plausible in that the treatment teachers may have been motivated by the new approach, which is an advantage that the business-as-usual teachers did not have.

Future Research

The results provide "proof of concept" (NCTM, 2007, p. 2) support for the efficacy of SBI. Focusing on the mathematical problem structure via schematic diagrams within the context of explicit instruction in problem solving and metacognition strategies, and encouraging students to employ multiple solution methods was more than or as effective as control instruction on mathematical problem solving. These conclusions and limitations of the study suggest several possibilities for future research. First, we intend to investigate the sustainability of SBI when implemented by SBI-experienced teachers as compared to SBI-novice teachers, which could also address a novelty effect of implementing a new approach. Although our previous studies show the promise of the SBI intervention when teachers implement SBI immediately after receiving training on its use, it is less clear whether teachers will persist in implementing SBI with fidelity in subsequent years. Second, we will replicate the current study in a different geographic location within the U.S. in a high needs urban district that includes substantial diversity in the student population. We expect the forthcoming replication to not only speak to the generalizability of SBI effects but also increase the between-classroom variance to better examine any moderating effects of SBI or teacher-classroom characteristics on the relationships between student-level background variables and mathematical problem solving.

SBI is a multicomponent intervention, and evidence from the fidelity of implementation data confirmed that SBI teachers required students to identify the problem type as well as discuss the similarities and differences between previously solved and new problems, identify relevant information to represent in a diagram, solve problems by estimating the answer, use multiple solution strategies, and check whether the answer makes sense, significantly more than control teachers. However, the scope of the current study did not include an examination of which features were most influential in student success on proportional reasoning suggesting that future research is needed to determine why SBI is effective. For example, a study that compares SBI as detailed in this study with SBI without the metacognitive strategy training is needed to examine the effects of metacognitive strategy training.

Another area of future research would be to examine the influence of longer duration of SBI to address the issue of transfer effects. Wagner (2006) argues that transfer is "the incremental growth, systematization, and organization of knowledge resources that only gradually extend the span of situations in which a concept is perceived as applicable" (p. 10). A study in which participants are randomly assigned to receive SBI for different durations (e.g., 6 weeks vs. 8 weeks) is needed to examine whether there is a transfer effect favoring longer duration of SBI. In addition, the SBI intervention may need to be revised to not only provide more general mathematics applications that include priming students to focus on the similarities and differences between new domains (e.g., algebra) and previously learned topics (e.g., ratios and proportion) to make explicit connections to content outside of the instructional domain, but also cover other content areas across the school year. It is also important to examine the long-term effects of SBI for students who have mastered the major SBI components in the context of proportional reasoning. That is, does mastery of specific SBI components prepare students to succeed in more advanced mathematics (e.g., algebra, geometry) and result in transfer of content learned (e.g., ratios and proportions) to other content areas (e.g., science)?

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Table 1.

Participant Demographic Information by Treatment

		SBI				Control				Total			
		n	%	*M*	*SD*	*n*	%	*M*	*SD*	*n*	%	*M*	*SD*
Student Information													
Age				12.80	0.37			12.77	0.35			12.78	0.36
Sex	Female	456	46.7			497	49.5			953	48.1		
	Male	499	51.1			472	47.0			971	49.0		
	Missing	22	2.3			35	3.5			57	2.9		
Race	Asian	61	6.2			64	6.4			125	6.3		
	Black	85	8.7			81	8.1			166	8.4		
	Hispanic	67	6.9			77	7.7			144	7.3		
	White	739	75.6			747	74.4			1486	75.0		
	Missing	25	2.6			35	3.5			60	3.0		
FRL	Yes	397	40.6			388	38.6			785	39.6		
	No	555	56.8			581	57.9			1136	57.3		
	Missing	25	2.6			35	3.5			60	3.0		
LEP	Yes	63	6.4			53	5.3			116	5.9		
	No	889	91.0			916	91.2			1805	91.1		
	Missing	25	2.6			35	3.5			60	3.0		
SpEd	Yes	107	11.0			82	8.2			189	9.5		
	No	845	86.5			887	88.3			1732	87.4		
	Missing	25	2.6			35	3.5t			60	3.0		
Location	Rural	551	56.4			520	51.8			1071	54.1		
	Suburban	339	34.7			335	33.4			674	34.0		
	Urban	87	8.9			149	14.8			236	11.9		

		SBI				Control				Total			
		n	%	*M*	*SD*	*n*	%	*M*	*SD*	*n*	%	*M*	*SD*
Teacher Information													
Sex	Female	26	65.0			29	69.0			55	67.1		
	Male	14	35.0			13	31.0			27	32.9		
Location	Rural	23	57.5			23	54.8			46	56.1		
	Suburban	14	35.0			13	31.0			27	32.9		
	Urban	3	7.5			6	14.3			9	11.0		
Math courses taken				8.60	3.77			8.70	4.20			8.65	3.97
Education courses taken				4.24	4.81			2.87	2.65			3.54	3.89
Years experience teaching math				11.95	6.38			12.42	7.02			11.93	6.35
PD hours in math				24.88	30.94			23.65	17.61			24.25	24.86

Note. SBI = schema-based instruction; PD = professional development; FRL = students eligible for free or reduced priced lunch; LEP = limited English proficiency; SpEd = students receiving special education services; Total student *N* = 1981 but some analyses were based on a smaller sample size because of missing data.

Table 2.

Descriptive Statistics for Measures by Treatment

	SBI			Control			Total		
	n	*M*	*SD*	*n*	*M*	*SD*	*n*	*M*	*SD*
PPS pretest	956	13.29	4.30	978	13.74	4.56	1934	13.52	4.44
PPS posttest	943	16.52	5.27	955	15.09	4.92	1898	15.80	5.15
PPS delayed	931	16.07	5.09	916	15.22	5.02	1847	15.65	5.07
GMADE pretest	952	13.23	3.76	974	13.44	3.95	1926	13.33	3.86
GMADE posttest	944	14.22	4.21	942	14.02	4.41	1886	14.12	4.31

Note. SBI = schema-based instruction; PPS = proportional problem solving; GMADE = Group Mathematics Assessment and Diagnostic Evaluation. All test statistics are based on the total number of items correct.

Table 3.

Quality of Instruction and Fidelity of Implementation by Treatment

	SBI			Control						
Measure	*n*	*M*	*SD*	*n*	*M*	*SD*	*t*	*df*	*p*	*ES*
Quality of Instruction										
Sets lesson purpose	40	2.63	0.49	40	2.60	0.84	0.16	78	.871	0.04
Provides lesson closure	40	1.08	0.83	40	1.08	0.69	0.00	78	.999	0.00
Classroom management	40	2.88	0.34	40	2.78	0.66	0.86	78	.395	0.19
Mathematical errors	40	2.93	0.47	40	2.83	0.55	0.87	78	.386	0.20
Total score	40	9.50	1.28	40	9.28	1.71	0.66	78	.507	0.15
Fidelity of Implementation										
Identifies problem type	40	2.10	1.11	40	0.75	0.90	6.00	78	<.001	1.34
Problem similar/different	40	1.28	1.11	40	0.35	0.70	4.46	78	<.001	1.00
Identifies key information	40	2.50	0.75	40	1.95	0.85	3.08	78	.003	0.69
Estimates solution	40	2.18	1.04	40	0.18	0.55	10.80	78	<.001	2.40
Multiple solution strategies	40	1.93	0.89	40	1.35	0.86	2.94	78	.004	0.66
Provides complete solution	40	2.65	0.70	40	2.15	1.00	2.59	78	.011	0.58
Checks solution	40	1.70	0.91	40	0.70	0.94	4.83	78	<.001	1.08
Total score	40	14.33	3.86	40	7.43	3.00	8.82	78	<.001	2.00

Note. SBI = schema-based instruction; Effect size (ES) was calculated as the two conditions' mean difference divided by the pooled standard deviation (Hedges & Olkin, 1985). Here $\alpha = \frac{.15}{13} = .0115$ following the Dunn-Bonferonni procedure.

Table 4.

HLM Results for PPS Posttest

Fixed Effects	*B*	*SE*	*t*	*df*	*p*
Between-Student Model					
Sex	0.22	0.18	1.20	1799	.231
Asian	0.11	0.36	0.31	1799	.757
Black	-1.06	0.33	-3.23	1799	.002
Hispanic	-0.52	0.36	-1.44	1799	.150
Pretest	0.68	0.03	27.64	1799	<.001
Between-Classroom Model					
Intercept	15.35	0.79			
Treatment	1.70	0.32	5.38	72	<.001
Sex	-0.02	0.35	-0.05	72	.963
Math courses	-0.00	0.04	-0.11	72	.916
Edu. courses	-0.02	0.03	-0.59	72	.559
Yrs. experience	0.04	0.03	1.30	72	.197
PD hours	0.00	0.01	0.05	72	.963
FRL	-0.23	0.15	-1.47	72	.145
LEP	-0.19	0.14	-1.41	72	.163
Special edu.	-0.02	0.10	-0.24	72	.809
Random Effects	VC	*SD*	χ^2	*df*	*p*
Classroom	1.17	1.08	254.45	72	<.001
Student	12.50	3.54			

Note. PD = professional development; FRL = eligible for free or reduced price lunch; LEP = limited English proficiency; VC = variance component. Here $\alpha = .05$ for the test of the SBI effect and $\alpha = \frac{.10}{14} = .0071$ for tests of the remaining fixed effects.

Table 5.

HLM Results for PPS Delayed Posttest

Fixed Effects	*B*	*SE*	*t*	*df*	*p*
Between-Student Model					
Sex	0.17	0.21	0.79	1755	.430
Asian	0.37	0.40	0.94	1755	.350
Black	-0.60	0.40	-1.52	1755	.128
Hispanic	-0.38	0.36	-1.05	1755	.293
Pretest	0.67	0.03	26.72	1755	<.001
Between-Classroom Model					
Intercept	15.07	0.64			
Treatment	1.22	0.29	4.17	70	<.001
Sex	0.33	0.31	1.05	70	.296
Math courses	-0.01	0.04	-0.16	70	.877
Edu. courses	0.02	0.03	0.89	70	.374
Yrs. experience	0.04	0.02	1.86	70	.067
PD hours	-0.00	0.00	-0.40	70	.687
FRL	-0.22	0.13	-1.74	70	.087
LEP	-0.19	0.13	-1.47	70	.147
Special edu.	-0.02	0.10	-0.22	70	.828
Random Effects	VC	*SD*	χ^2	*df*	*p*
Classroom	0.76	0.87	178.24	70	<.001
Student	13.64	3.69			

Note. PD = professional development; FRL = eligible for free or reduced price lunch; LEP = limited English proficiency; VC = variance component. Here $\alpha = .05$ for the test of the SBI effect and $\alpha = \frac{.10}{14} = .0071$ for tests of the remaining fixed effects.

Table 6.

HLM Results for GMADE Posttest

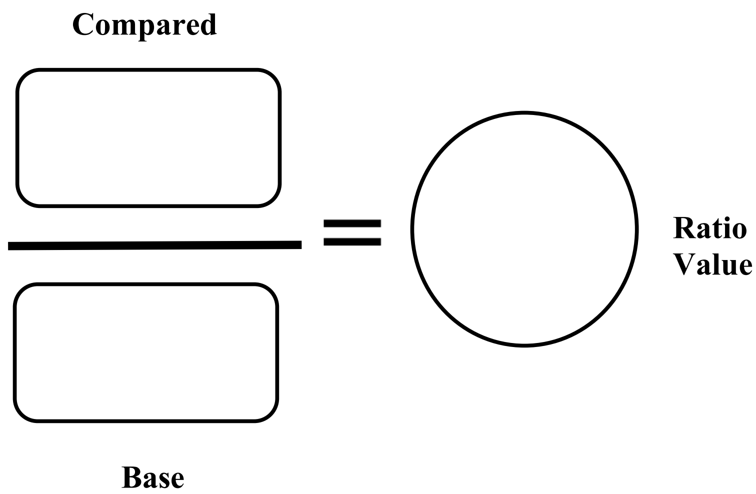
Fixed Effects	*B*	*SE*	*t*	*df*	*p*
Between-Student Model					
Sex	0.10	0.18	0.52	1786	.600
Asian	0.50	0.31	1.59	1786	.112
Black	-0.54	0.32	-1.72	1786	.086
Hispanic	-1.01	0.35	-2.87	1786	.005
Pretest	0.52	0.03	17.65	1786	<.001
Between-Classroom Model					
Intercept	14.50	0.57			
Treatment	0.39	0.30	1.31	72	.194
Sex	0.02	0.32	0.05	72	.960
Math courses	-0.05	0.03	-1.72	72	.089
Edu. courses	0.03	0.03	0.89	72	.378
Yrs. experience	0.07	0.02	3.37	72	.002
PD hours	-0.00	0.00	-0.60	72	.549
FRL	-0.21	0.12	-1.66	72	.102
LEP	-0.23	0.11	-2.09	72	.040
Special edu.	-0.14	0.10	-1.43	72	.156
Random Effects	VC	*SD*	χ^2	*df*	*p*
Classroom	0.80	0.90	208.95	72	<.001
Student	11.30	3.36			

Note. PD = professional development; FRL = eligible for free or reduced price lunch; LEP = limited English proficiency; VC = variance component. Here $\alpha = .05$ for the test of the SBI effect and $\alpha = \frac{.10}{14} = .0071$ for tests of the remaining fixed effects.

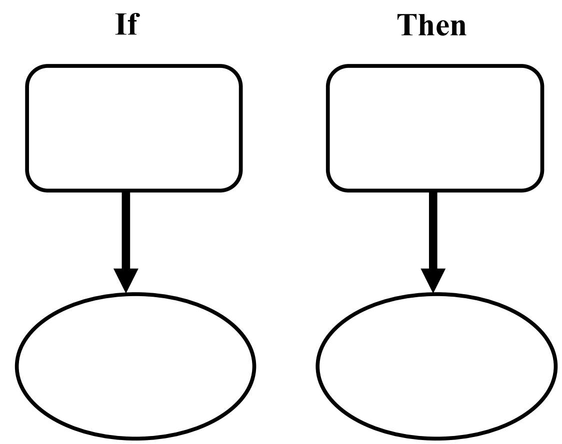
Figure Caption

Figure 1. Ratio, Proportion, and Percent of Change Diagrams

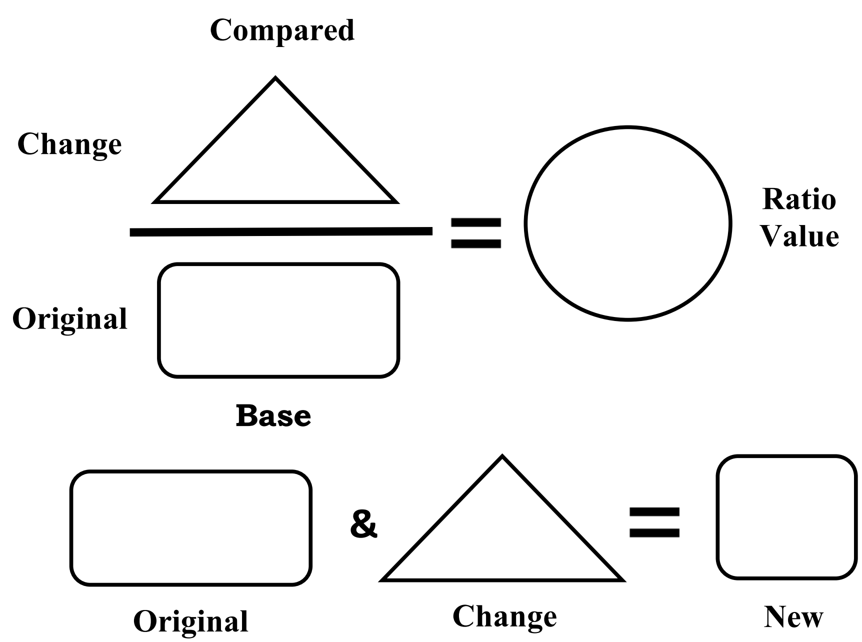
Ratio Diagram



Proportion Diagram



Percent of Change Diagram

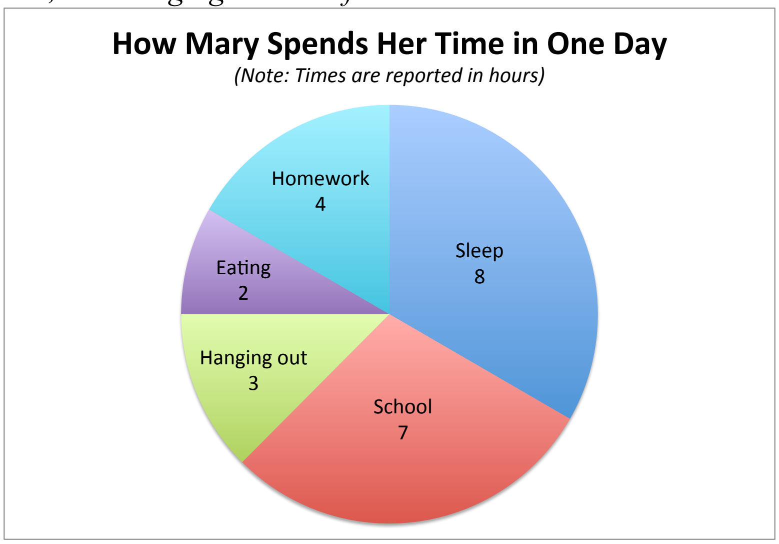


APPENDIX A

Sample Excerpts from Scripts for Teaching Percent Problem Solving

Fractions, Percents, and Decimals (Lesson 11)

Problem: The circle graph depicts how Mary spends her time every day. It shows how many hours per day Mary spends on each of the following activities: sleeping, eating, going to school, completing homework, and hanging out with friends.



A. What fraction of Mary's time is spent sleeping and doing homework?
B. What fraction of Mary's total time is spent ***not*** going to school?
C. When added together, which two activities did Mary spend 25% of her total time on?

Teacher: Let's solve the problem using information in this circle graph. This graph tells how many hours Mary spends on each of the following activities: sleeping, going to school, eating, doing homework, and hanging out with friends. If we want to figure out a fractional expression of the amount of time that Mary spends sleeping and doing homework, we should first calculate the total number of hours, which is the base quantity. What is the total number of hours Mary spends on the different activities? Explain.

Students: 24 hours. I added up all the hours Mary spends sleeping (8 hours), going to school (7 hours), doing homework (4 hours), eating (2 hours), and hanging out with friends (3 hours).

Teacher: Right! 24 hours is the base quantity. That is also the number of hours in one day. And from the graph, we know that Mary spent 8 hours sleeping and 4 hours doing homework. So 8 + 4 =12 hours is the compared quantity. We can then write the ratio of time spent sleeping and doing homework to the total amount of time as 12:24. A ratio of 12:24 can be written in fraction form as $\frac{12}{24}$. When we simplify the fraction $\frac{12}{24}$, what do we get?

Students: $\frac{1}{2}$.

Teacher: How can we write $\frac{1}{2}$ as a percent?

Students: We know that we can write $\frac{1}{2}$ in many different ways. One fraction that is equivalent to $\frac{1}{2}$ is $\frac{50}{100}$. For $\frac{50}{100}$, the denominator is 100, so we can write $\frac{50}{100}$ as 50%.

Teacher: Next, we need to figure out what fraction of Mary's total time is spent not going to school. *(Think aloud)* Let's see … we know from just solving the question (11.6A) about the amount of time Mary spent sleeping and doing homework that the base quantity (total time spent on all activities) is 24. We need to find the compared quantity, which is the time spent not going to school. What is this amount? Explain.

Students: 17 hours. With the exception of the hours spent going to school (i.e., 7 hours), I added the hours spent sleeping (8 hours), doing homework (4 hours), eating (2 hours) and hanging out with friends (3 hours). So 8 + 4 + 2 + 3 = 17.

Teacher: So, 17 is our compared quantity, which is the time not spent going to school, and 24 is the base quantity (i.e., the total time spent going to school and not going to school). We need to write this as a fraction. How would you write the amount of time spent not going to school to the total amount of time as a fraction? Explain.

Students: We would write the compared quantity (numerator) over the base quantity (denominator) to get $\frac{17}{24}$.

Teacher: Great! Now, $\frac{17}{24}$ cannot be simplified, so Mary spends $\frac{17}{24}$ of her time not going to school. Now, we are ready to solve the next question about which two activities Mary spends 25% of her time doing in one day. Let's use these 4 steps (DISC) to solve this problem. To discover the problem type, I will read, retell, and examine information in the problem to recognize the problem type. (Read and retell the problem.) Now, I will ask myself if this is a percent problem. How do I know it is a percent problem? It's a percent problem, because it tells about the percent of time Mary spends on a specific activity every day. The percent relationship describes a part-to-whole (time Mary spends doing each activity out of the total time of 24 hours) comparison.

Now I will ask if this problem is different from/similar to another problem we have already solved. (Remind students of the ratio problems just solved.) It is similar to ratio problems we solved earlier, because it compares a part (i.e., the number of hours) to the whole (total hours). However, it is different from those ratio problems, because the relation between the part-to-whole is expressed as a percent.

(Point to Step 2 on the checklist.) Now we are ready to identify information in the problem to represent in a diagram(s). Let's write the percent as a ratio in the diagram. Remember, a percent is any number that is compared to 100. The percent sign is just another way of saying that this is a ratio with 25 as the compared quantity and 100 as the base quantity. So 25% is the same as 25:100 or $\frac{25}{100}$. It is also the same as 0.25, because 25 divided by 100 is 0.25. All three of these representations mean 25 (parts) out of 100 (whole). Let's write $\frac{25}{100}$ for the ratio value in the diagram. Let's write "number of hours Mary spent on two activities" for the compared and "total hours in the day" for the base quantity in the diagram. We reasoned earlier that "24 hours" ("total hours in the day") is the base quantity.

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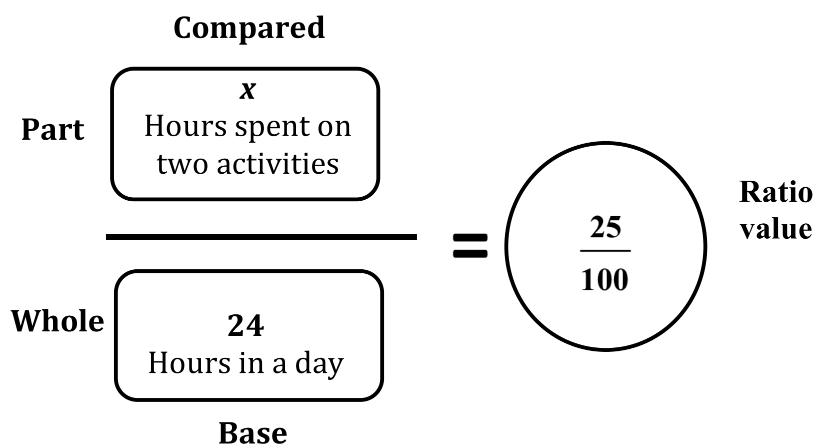

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Teacher: (Point to Step 3 on the checklist.) Now we are ready to solve the problem. Before we solve the problem, let's first come up with an estimate for the answer. It is hard to come up with an estimate, but I do know a few things about the answer. I know that the answer is less than 24, because the total number of hours (whole) is 24, and the time Mary spent on the two activities is less than 100% of 24. And I know that 25% is less than 50%, and 50% or $\frac{1}{2}$ is 12. So the answer should be less than 12. So looking at the circle graph, my estimate is that the answer is either 1, 2, 3 or 6 hours.

Next, I will translate the information in the diagram into a math equation. From the diagram, we can set up the math equation to look like this (point to the equation.):

$$\frac{x \text{ hours spent on two activities}}{24 \text{ total hours}} = \frac{25}{100}$$

Now I need to figure out what strategy to use to solve for the number of hours spent on the activity (which is x). You learned several strategies (i.e., unit rate, equivalent fractions, or cross multiplication) to solve ratio and proportion problems. What strategy would you use to solve for x in the problem? Explain.

Students: *Unit rate strategy, because it works best with the numbers in the problem.*

Teacher: Solve for the unknown or *x* in the equation using the unit rate strategy we identified in the plan step. When you use unit rate strategy, how do you set up the problem? Show all your work and explain.

Students:

$$\downarrow \frac{x \text{ hours spent on two activities}}{24 \text{ total hours}} = \frac{25}{100} \downarrow$$

First, ask 25 times what number gives 100. The answer is 4 (i.e., $25 \cdot 4 = 100$). Then, we multiply x by 4 to get $4x = 24$. So $x = 24 \div 4$, which is 6.

Teacher: So, 6 hours is our compared quantity. *(Point to the circle graph.)* From the circle graph, which two activities when you add together did Mary spend 6 hours on?

Students: *Mary spent 6 hours doing homework and eating.*

Teacher: Good. Let's write 6 for "*x*" in the diagram and write the complete answer on the answer line. What is the complete answer?

Students: *Mary spent 6 hours doing homework and eating.*

Teacher: Good. What do you do next? *(Point to Step 4.)*

Students: *Check if the answer makes sense.*

Teacher: To check the solution, what must you do first?

Students: *Look back to see if our estimate in Step 3 is close to the exact answer.*

Teacher: We estimated our answer to be less than 12 hours. The answer to this problem is 6. So, your estimate did a nice job of giving us a ballpark sense of what the answer should be. Now check to see if the answer makes sense. Does 6 seem right? Explain.

Students: *We know that if 24 hours represent 100% of the total time in Mary's day, then 6 hours going to school out of 24 hours seems right. The answer of 6 hours doing homework and eating seems right, because this is less hours (6) than the total number of hours in a day (24).*

Teacher: You can also check the ratio 6:24 to see if this value is equal to the ratio 25:100. When you simplify both ratios *(i.e., using your calculator, divide the numerator by the denominator)*, you get $\frac{6}{24}$ (0.25), which tells me that they are equivalent.

Solving Percent Word Problems: Percent of Change (Lesson 14)

Problem: There is great variation between day and night temperatures in summer in the deserts of Rajasthan, India. Find the percent of change in degrees Fahrenheit in the deserts of Rajasthan from 120°F in the daytime on May 5, 2006, to 84°F that night.

Teacher: Let's use these 4 steps (DISC) to solve this problem. To discover the problem type, I will read, retell, and examine information in the problem to recognize the problem type. *(Read and retell the problem.)* How do you know if this is a percent problem?

Students: It's a percent problem, because it tells about the percent of change in degrees Fahrenheit from day to night.

Teacher: Is this problem different from/similar to another problem you have solved? This problem is similar to the percent of change problems (e.g., plant growth, allowance, weight loss) we solved yesterday, because it compares the change to the original amount, but is different in that it describes the percent of change in degrees Fahrenheit from day to night. In this temperature problem, we are given the degrees Fahrenheit during the day (original) and night (new) and asked to find the percent of change in degrees Fahrenheit from day to night in Rajasthan, India.

(Point to Step 2 on the checklist.) Now we are ready to identify information in the problem to represent in the diagrams.

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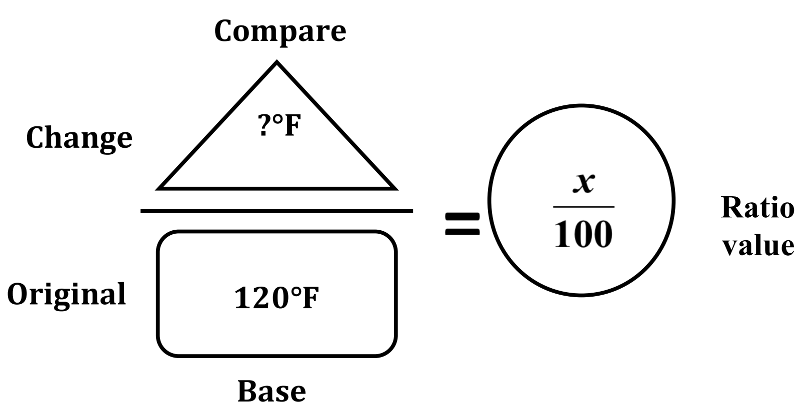

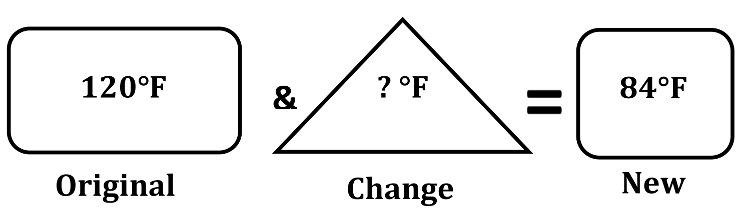

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Teacher: Now we are ready to solve the problem. To solve the problem, we need to come up with an estimate for the answer. What is the estimate of the percent of change in degrees Fahrenheit in the deserts of Rajasthan from 120°F in the daytime on May 5, 2006 to 84°F that night?

Students: It is hard to come up with an estimate, but I do know a few things about the answer. I know that the degrees Fahrenheit changed about 30 or 40 degrees, from 120° down to 84°. If the degrees Fahrenheit had changed 60°, this is half of 120°, so this would be 50% change. So I would estimate the answer to be close to 50%, perhaps larger than 25%. So I think the percentage of change will be greater than 25%, but less than 50%.

Teacher: Excellent! What do you do next?

Students: Translate the information in the diagram into a math equation.

Teacher: Before we translate the information, look at both diagrams and tell me what we don't know.

Students: We don't know the change amount, and we don't know the percent of change.

Teacher: Great. So which one should we figure out first? Why?

Students: You can't figure out the percent of change until you find the change amount. So, we first need to solve for the change amount.

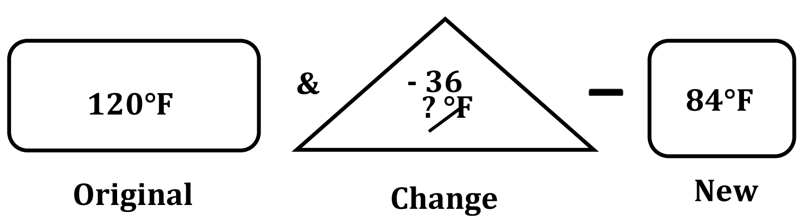
Teacher: How would you solve for the change amount.

Students: We can solve for the "?" in the change diagram to find the change amount.

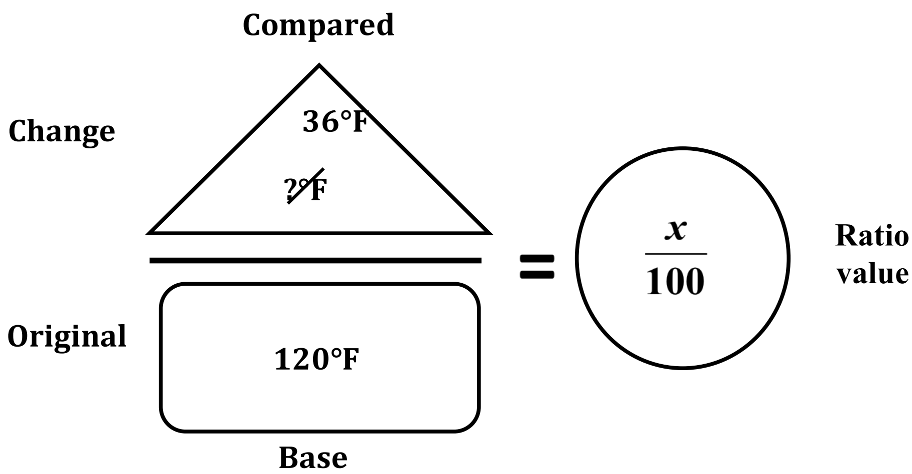
Teacher: What do you have to do with 120 to get 84? Explain

Students: Subtract 36, because there is a decrease in degrees Fahrenheit.

Teacher: Cross out the "?" and write −36 in the triangle for the Change diagram. This tells us that the change is a decrease.



Now remember that this change amount appears in both diagrams, so let's also cross out the "?" in the Ratio diagram and write 36. Let's look at the Ratio diagram, which shows a model of the problem situation. *(Point to the diagram below.)* What must you solve for in this problem?



Students: *The percent of change.*

Teacher: You can solve for x and find the percent of change (i.e., percent of decrease) using the Ratio diagram. Now, you can translate the information in the diagram into a math equation. From the diagram, show how you would set up the math equation.

Students: $\frac{36°\text{F}}{120°\text{F}} = \frac{x}{100}$

Teacher: Now you need to figure out what strategy (e.g., cross multiplication, unit rate, equivalent fractions) to use to solve for x. What strategy would you use to solve for x? Explain.

Students: *Cross multiplication, because it works well with the numbers in the problem.*

Teacher: Now solve the math equation and show all the steps. *(Point to the equation.)*

$$\frac{36°\text{F}}{120°\text{F}} = \frac{x}{100}$$

Students: *The first step is to cross multiply, which yields:*

$$120 \cdot x = 36 \cdot 100$$

or

$$120x = 3600$$

The second step is to solve for x, which is the percent of change in degrees Fahrenheit from day to night. To solve this equation for x, we need to divide both sides of the equation by 120. So 3600 ÷ 120 is 30, which is the answer. I solved for x, which is 30 (i.e., a 30% decrease in degrees Fahrenheit in the Rajasthan desert from day to night).

Teacher: Now we know the percent of change (x), which is 30. Write 30 for "x" in the diagram. The problem asks us to find the percent of change. Does anyone know how to write $\frac{30}{100}$ as percent?

Students: 30%.

Teacher: So what is the complete answer to this problem?

Students: The percent of change in degrees Fahrenheit is 30%

Teacher: That's right! This percent of change is a decrease in degrees Fahrenheit. Write "The percent change in degrees Fahrenheit from day to night is 30%" on the answer line. Writing the answer in complete form helps to describe exactly what your answer means. What do you do next? *(Point to Step 4.)*

Students: Check if the answer makes sense.

Teacher: To check the solution, what must you do first?

Students: Look back to see if our estimate is close to the exact answer

Teacher: We estimated our answer to be between 25% and 50%. Let's check to see if our estimate is close to the exact answer. What is the correct answer?

Students: 30%.

Teacher: So, your estimate (between 25% and 50%) is a good prediction because it is in the ballpark of the exact answer (30%). What do you do next?

Students: Check if the answer makes sense.

Teacher: Does 30% seem right? Explain.

Students: We know that if 120°F represents 100%, then the change in temperature of 36°F seems right, because the change involves a decrease of the original temperature (120°F).

Teacher: You can also check the ratio 36:120 to see if this value is equal to the ratio 30:100. When you use a calculator and divide the numerator by the denominator, you get 0.30, which tells me that they are equivalent. Let's review this percent of change problem. What is this problem called? Why?

Students: It is a percent change problem, because it involves a comparison of the change amount to the original amount. The new amount (degrees Fahrenheit at night) is a decrease of the original amount (degrees Fahrenheit in the daytime), so the change involves a decrease.

APPENDIX B

Sample Items from the Proportional Problem Solving Test

1. The weight of an object on the Moon is $\frac{1}{6}$ the weight of that object on the Earth. An object that weighs 30 pounds on Earth would weigh how many pounds on the Moon?

   A. 10 pounds
   B. 11 pounds
   C. 12 pounds
   D. 13 pounds

2. At the school carnival, Carmen sold 3 times as many hot dogs as Shawn. The two of them together sold 152 hot dogs. How many hot dogs did Carmen sell?

   A. 38
   B. 51
   C. 114

3. A club held a car wash and washed 21 cars. If the club raised $84, how much did it charge per car?

   A. $0.25
   B. $4.00
   C. $5.00
   D. $4.50

4. If there are 300 calories in 200 g of a certain food, how many calories are there in a 40 g portion of this food?

   A. 60
   B. $26\frac{2}{3}$
   C. 6
   D. 140

5. A machine uses 2.4 liters of gasoline for every 30 hours of operation. How many liters of gasoline will the machine use in 100 hours?

   A. 7.2
   B. 8.0
   C. 8.4

6. Marcel's drawing of an ant is $4\frac{1}{2}$ inches long. His drawing is 12 times the ant's actual size. How long is the actual ant?

A. $\frac{1}{3}$ inch
B. $\frac{3}{8}$ inch
C. $\frac{1}{2}$ inch
D. $\frac{8}{3}$ inch

7. Some classmates compared their scores on a recent math test.

- Molly answered 15 out of every 20 questions correctly.
- Brittany answered 7 out of every 8 questions correctly.
- Desiree answered 7 out of every 10 questions correctly.
- Nick answered 4 out of every 5 questions correctly

Which student answered more than 80% of the questions correctly?

A. Molly
B. Brittany
C. Desiree
D. Nick

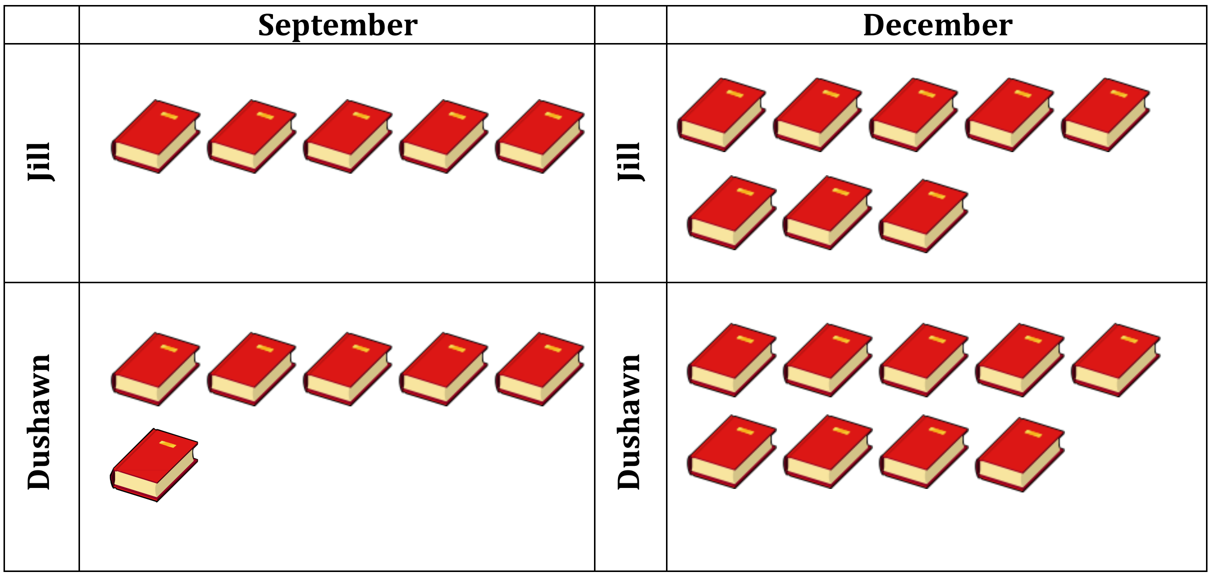
8. If the price of a can of beans is raised from 50 cents to 60 cents, what is the percent increase in the price?

A. 83.3%
B. 20.0%
C. 16.7%
D. 10.0%

9. Taylor was earning an income of $1,000 a week. Then his income was reduced by 10%. Two months later, his income increases by 10%. How much is Taylor earning, in dollars, after his income increases?

A. $990
B. $1,000
C. $1,100

10. Jill and Dushawn joined a book club in middle school. The picture below shows how many books Jill and Dushawn each read in September and December. In September, Jill read 5 books and Dushawn read 6 books. In December, Jill read 8 books and Dushawn read 9 books.



Dushawn thinks that the amount of books he and Jill read from September to December is the same**. Use mathematics to explain how Dushawn might have justified his claim**.

Jill thinks that she has read more books than Dushawn from September to December. **Use mathematics to explain how Jill might have justified her claim.**